Phase retrieval: physics and algorithms for light at various states of coherence

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The importance of phase information, as well as the difficulty of its retrieval at high frequencies, are both undisputed. This tutorial will provide a survey of the basics of phase retrieval as well as some recent developments on physical implementations and inversion algorithms, especially those utilizing sparsity priors.

Assuming the illumination is temporally quasi-coherent, there are two cases of interest: spatially coherent, and spatially partially coherent. In the spatially coherent case, interferometric/holographic techniques apply best; recently, a variety of new methods have also come to the fore, and they can be classified as: (1) propagation-based, and (2) phase-space based. The most common realization of propagation-based methods inverts the transport-of-intensity equation, so that phase is obtained from the intensity and its derivative with respect to the propagation variable $z$. Phase-space methods involve the use of wavefront sensors / lenslet arrays, as well as ptychography—even though the latter seldom employs the phase space itself in the computation of the inverse; instead, iterative inversion methods are preferred.

Spatially partially coherent light is interesting for a variety of reasons, not the least of which are that it tends to cause less damage to biological samples and it has more spatial degrees of freedom (three, whereas coherent light is two-dimensional.) However, interferometric methods suffer from low contrast when the light is partially coherent; moreover, there is a methodological problem: “phase” is technically not well defined. The latter problem is solved by instead mapping the measurement onto the Optical Path Length (OPL) of light through the sample and recognizing that OPL is usually the sought-after quantity, even for coherent light.

OPL retrieval then becomes a problem of retrieving the correlation function of partially coherent light from intensity measurements. Recovery methods may also be classified as interferometric/propagation-based/phase-space-based, and there are of course hybrids, such as phase-space tomography. Of particular interest for the inverse problem is the so-called coherent mode decomposition property of correlation functions that follows from Mercer’s theorem:

$$J(r_1, r_2) = \sum_n c_n \phi_n(r_1) \phi_n^*(r_2),$$

where $J$ is the correlation function, $r_1$, $r_2$ are spatial coordinates, $c_n$ are complex constants, and $\phi_n$ are the “coherent modes.” In many cases, the argument can be made that very few of the $c_n$ coefficients are non-zero. Then, sparsity-based algorithms (“compressive sensing”) can be used effectively to regularize the reconstruction.

Extensive references to the literature will be given during the talk.